Knowsley Junior School

Calculation Policy

Introduction

The following calculation policy has been adapted to meet the requirements of the national curriculum 2014 for the teaching and learning of mathematics. It has been drawn up as a result of staff discussion, and its implementation is the responsibility of all teaching staff. The responsibility for monitoring and review rests with the mathematics leader, Elizabeth Oliver.

Aims and Objectives

At Knowsley Junior School we believe that children should be introduced to the processes of calculation through practical, oral and mental activities, linked to real life or lifelike experiences. As children begin to understand the underlying ideas, they develop ways of recording to support their thinking. Children learn how to use models and images, such as empty number lines, to support their mental and informal written methods of calculation. As children's mental methods are strengthened and refined, so too are their informal written methods. These methods become more efficient and succinct and lead to efficient written methods that can be used more generally.

By the end of Year 6, children are equipped with mental and written methods that they understand and can use correctly. When faced with a calculation, children are able to decide which method is most appropriate and have strategies to check its accuracy. At whatever stage in their learning, and whatever method is being used, it must still be underpinned by a secure and appropriate knowledge of number facts, along with those mental skills that are needed to carry out the process and judge if it was successful.

The overall aim is that when children leave Knowsley Junior School they:

- Have a secure knowledge of number facts and a good understanding of the four operations.
- Are able to use this knowledge and understanding to carry out calculations mentally and to apply general strategies when using one-digit and two-digit numbers, and particular strategies to special cases involving bigger numbers.
- Make use of diagrams and informal notes to help record steps and part answers when using mental methods that generate more information than can be kept in their heads.
- Have an efficient, reliable, compact written method of calculation for each operation, that they can apply with confidence when undertaking calculations that they cannot carry out mentally.
- Are able to use and apply their mathematical skills in practical, problem-solving contexts.

Key Strategies

a. Hands-on learning is important

Provide appropriate, practical equipment for children to use and manipulate, to help them to explore how and why things work and to learn to visualise, describe and represent what is in front of them.

b. Seeing mathematics through models and images supports learning

Help children to see how mathematics can be represented through physical objects, pictures/diagrams such as place-value cards, counting sticks, number lines, representations of fractional parts.

c. Talking mathematics clarifies and refines thinking

Give children the vocabulary and language of mathematics; provide activities and time for them to discuss mathematics, using this language. Teach the children the precision of language, e.g. using 'prism' correctly.

d. Make mathematics interesting and linked to real-life or life-like situations

Try to provide real life or lifelike contexts, across the curriculum, for using and applying mathematical skills so that maths becomes 'real' to them.

e. Learning from mistakes should build up children's confidence

Look out for mistakes and encourage children to recognise that making mistakes is something everyone does. Show children common errors and get them to identify and correct them. Encourage children to work with others and share their work.

Adapted from DCSF Securing Levels materials, 2009

Teaching and Learning

Mental Methods of Calculation

Oral and mental work in mathematics is essential, particularly so in calculation. Early practical, oral and mental work must lay the foundations by providing children with a good understanding of how the four operations build on efficient counting strategies and a secure knowledge of place value and number facts. Later work must ensure that children recognise how the operations relate to one another and how the rules and laws of arithmetic are to be used and applied. Ongoing oral and mental work provides practice and consolidation of these ideas. It must give children the opportunity to apply what they have learned to particular cases, exemplifying how the rules and laws work,

and to general cases where children make decisions and choices for themselves.

The ability to calculate mentally forms the basis of all methods of calculation and has to be maintained and refined. A good knowledge of, or a feel, for numbers is the product of structured practice and repetition. It requires an understanding of number patterns and relationships developed through directed enquiry, use of models and images and the application of acquired number knowledge and skills.

Calculators should not be used as a substitute for good written and mental arithmetic. They should be introduced near the end of Key Stage 2 to support pupils' conceptual understanding and exploration of more complex number problems.

Talking

There is evidence that peer interactions are a main facilitator for socio-cognitive development and their performance in maths tasks, i.e. pupils learn better when talking and working in a group. A classroom culture of questioning in which pupils learn from shared discussions with teachers and peers promotes pupils' progress.

Teachers should help pupils to feel free and confident to comment and to suggest strategies even if there are errors in calculations. This allows for all pupils to reflect and suggest more appropriate methods. Teachers must value all contributions and be willing to change their own minds in the light of what pupils say, as this models a problem-solving approach. Tasks should be planned so there are opportunities for pupils to communicate their evolving understanding.

We value the communication between teachers and pupils, pupils and their peers. When children feel confident to communicate their ideas and discuss their findings openly this improves their level of understanding. The national curriculum states that 'the quality and variety of language that pupils hear and speak are key factors in developing their mathematical vocabulary and presenting a mathematical justification, argument or proof.'

Vocabulary

Vocabulary builds as the children progress through the school and the vocabulary highlighted for each year group are the most frequently used words.

Written Methods of Calculation

By the end of Key Stage 2, the great majority of children should be able to use an efficient written method for each operation with confidence and understanding. This guidance promotes the use of what are commonly known as 'standard' written methods; methods that are efficient and work for any calculations, including those that involve whole numbers or decimals. They are compact and consequently help children to keep track of their recorded steps. Being able to use these written methods gives children an efficient set of tools that they can use when they are unable to carry out the calculation in their heads. We want children to know that they have a reliable, written method to which they can turn when the need arises.

In setting out these aims, the intention is that we adopt greater consistency in our approach to calculation. The challenge is for our teachers to determine when their children should

move on to a refinement in the method and become confident and more efficient at written calculation. Children should be equipped to decide when it is best to use a mental or written method, based on the knowledge that they are in control of this choice as they are able to carry out both methods with confidence.

Choosing the Appropriate Strategy

Recording in mathematics (and during calculation in particular) is an important tool both for fostering an understanding of ideas and for communicating those ideas to others. A useful written method is one that helps children carry out a calculation and can be understood by others. Written methods are complementary to mental methods and should not be seen as separate from them. The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence.

Age-Related Expectations

The calculation policy is organised according to age-related expectations, as set out in the national curriculum 2014, however it is vital that pupils are taught according to the stage that they are currently working at, being moved onto the next level as soon as they are ready, or working at a lower stage until they are secure enough to move on. Teachers are required to make this professional judgement.

Signed Chair of Governors ……

Date ……../……../……..

Review ……../……../……..

WRITTEN METHODS FOR ADDITION

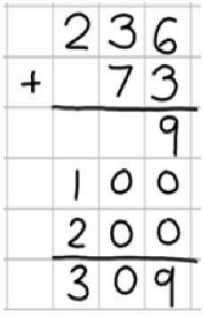
Year 3

Pupils should be taught to:

- Find 10 or 100 more than a given number (to at least 1000).
- Add numbers mentally, including
  digit and ones
  3 digit and tens
  3 digit and hundreds.
- Add numbers with up to three digits, using formal written methods of column addition.
- Estimate the answer to a calculation and use inverse operations to check answers.
- Solve problems, including missing number problems, using number facts, place value and more complex addition and subtraction.
- Continue to practise a wide range of mental addition strategies, i.e. number bonds, adding the nearest multiple of 10, 100 and adjusting, using near-doubles, partitioning and recombining.

Year 3 *Add numbers with up to 3 digits. Introduce expanded column addition.*

To add up to three digit numbers in vertical layout, write the numbers in columns adding the ones first:



Children who are very secure and confident with 3-digit expanded column addition should be moved onto the compact column addition method (below), being introduced to carrying for the first time. Compare the expanded method to the compact column method to develop an understanding of the process and the reduced number of steps involved.

47	258	366
+ 76	+ 87	+ 458
123	345	824
11	11	11

Remind pupils that the actual value is, for example, 'four tens add seven tens' in the first example, rather than '4 add 7'. 'Carry' numbers underneath the bottom line.

Column addition remains efficient when used with larger whole numbers and decimals. Once learned, the method is quick and reliable.

Key Vocabulary: *add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, plus, addition, column, tens boundary, hundreds boundary, increase, vertical, carry, expanded, compact.*

Year 4

Pupils should be taught to:

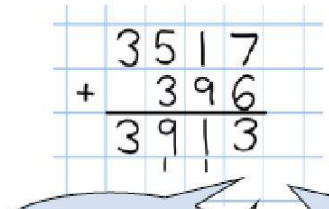
- Find 1000 more than a given number.
- Solve number and practical problems that involve addition with increasingly large positive numbers.
- Add numbers with up to 4 digits using the formal written methods of column addition and subtraction where appropriate.
- Estimate and use inverse operations to check answers to a calculation.
- Solve two-step addition problems in contexts, deciding which operation to use and why.
- Add fractions with the same denominator.
- Count forwards in hundredths.
- Solve simple measure and money problems involving fractions and decimals to two decimal places.

Year 4 *Add numbers with up to 4 digits.*

Move from expanded addition to the compact column method, **adding units first**, and 'carrying' numbers **underneath** the calculation. Also include money and measures contexts.

e.g. 3517 + 396 = 3913

Introduce the **compact column addition** method by asking children to add the two given numbers together using the method that they are familiar with (expanded column addition—see Y3). Teacher models the compact method with carrying, asking children to discuss similarities and differences and establish how it is carried out.



Add **units** first.

'Carry' numbers **underneath** the bottom line.

Reinforce correct place value by reminding them the actual value is 5 hundreds add 3 hundreds, **not 5 add 3**, for example.

Use and apply this method to money and measurement values.

Key Vocabulary: *add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, partition, plus, addition, column, tens boundary, hundreds boundary, increase, vertical, carry, expanded, compact, thousands, hundreds, digits, inverse.*

Year 5
Pupils should be taught to:

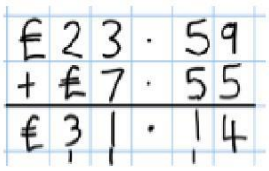
- Count forwards in steps of powers of 10 for any given number up to 1 000 000.
- Interpret negative numbers in context, count forwards with positive and negative whole numbers, including through zero.
- Add whole numbers with more than 4 digits, including using formal written methods (column addition and subtraction).
- Add numbers mentally with increasingly large numbers.
- Use rounding to check answers to calculations.
- Solve addition multi-step problems in contexts, deciding which operations and methods to use and why.
- Solve problems involving addition and all four operations, including understanding the meaning of the equals sign.
- Add fractions with the same denominator and denominators that are multiples of the same number.
- Solve problems involving measure, e.g. length, mass, volume, money using decimal notation, including scaling.
- Add fractions with the same denominator and denominators that are multiples of the same number.

Year 5 *Add numbers with more than 4 digits including money, measures and decimals with different numbers of decimal places.*

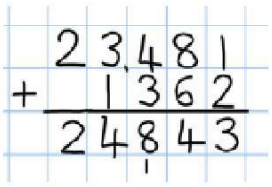
789 + 642 becomes

$$\begin{array}{r} 7\ 8\ 9 \\ +\ 6\ 4\ 2 \\ \hline 1\ 4\ 3\ 1 \\ \hline \quad 1\ 1 \end{array}$$

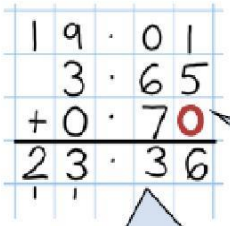
Answer: 1431



The decimal point should be aligned in the same way as the other place value columns, and must be in the same column in the answer.



Numbers should exceed 4 digits.



Pupils should be able to add more than two values, carefully aligning place value columns.

Empty decimal places can be filled with zero to show the place value in each column.

Say '6 tenths add 7 tenths' to reinforce place value.

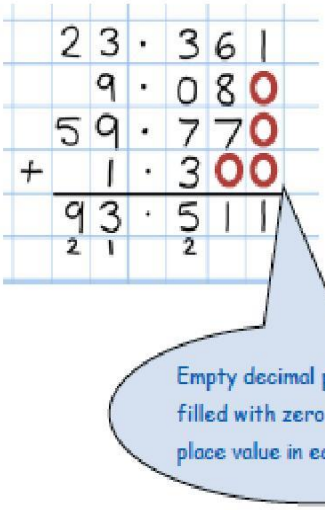
Key Vocabulary: *As for previous years + decimal places, decimal point, tenths, hundredths, thousandths.*

Year 6

Pupils should be taught to:

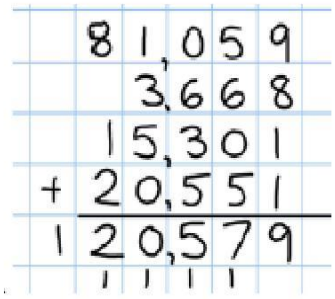
- Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why.
- Solve problems involving addition, subtraction, multiplication and division.
- Practise addition, subtraction, multiplication and division for larger numbers, using the formal written methods of column addition and subtraction, short and long multiplication/division.
- Perform mental calculations including mixed operations and increasingly large numbers and more complex calculations, using a range of mental strategies.
- Add fractions with different denominators and mixed numbers, using the concept of equivalent fractions.
- Use simple algebraic formulae.
- Generate and describe linear number sequences.

Year 6 *Add several numbers of increasing complexity.*



Adding several numbers with different numbers of decimal places (including money and measures):

- Tenths, hundredths and thousandths should be correctly aligned, with the decimal point lined up vertically including in the answer row.
- Zeros could be added into any empty decimal places, to show there is no value to add.



Adding several numbers with more than 4 digits.

Key Vocabulary: *As for previous years.*

WRITTEN METHODS FOR SUBTRACTION

Year 3

Pupils should be taught to:

- Find 10 or 100 less than a given number (to at least 1000).
- Subtract numbers mentally, including:
  - 3 digit and ones
  - 3 digit and tens
  - 3 digit and hundreds
- Subtract numbers with up to three digits, using formal written methods of column addition.
- Estimate the answer to a calculation and use inverse operations to check answers.
- Solve problems, including missing number problems, using number facts, place value and more complex addition and subtraction.
- Continue to practise a wide range of mental addition strategies, i.e. number bonds, adding the nearest and adjusting, using near-doubles, partitioning and recombining.

Year 3 *Subtract with 2 and 3-digit numbers. Introduce partitioned column subtraction method.*

STEP 1: introduce this method with examples where **no exchanging** is required.

89 – 35 = 54

80 + 9

- 30 + 5

50 + 4

When learning to 'exchange', explore 'partitioning in different ways' so that pupils understand that when you exchange, the **VALUE** is the same ie **72** = 70+2 = 60+12 = 50+22 etc. Emphasise that the **value hasn't changed**, we have just partitioned it in a different way.

STEP 2: introduce 'exchanging' through practical subtraction. Make the larger number with Base 10, then subtract 47 from it.

72 - 47

60 70 + 12

- 40 + 7

20 + 5 = **25**

Before subtracting '7' from the 72 blocks, they will need to exchange a row of 10 for ten units. Then subtract 7, and subtract 4 tens.

STEP 3: *Once pupils are secure with the understanding of 'exchanging', they can use the partitioned column method to subtract any 2 and 3-digit numbers.*

238 - 146 = 92

100

200 + 130 + 8

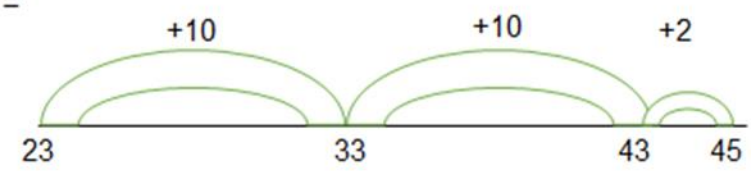
- 100 + 40 + 6

0 + 90 + 2

Counting on as a mental strategy for subtraction:

Subtract a 2 digit number by finding the difference and counting on:

45 - 23 =



10+10+2 = 22

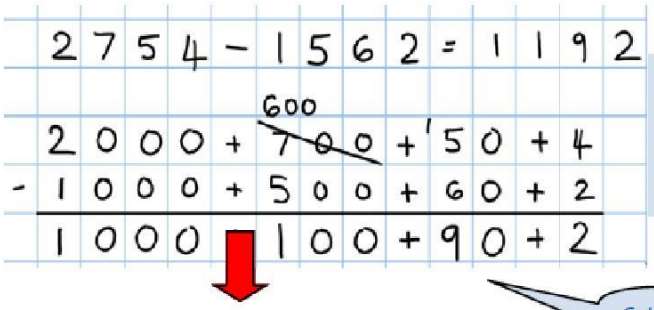
Key Vocabulary: *equal to, take, less, minus, subtract, leaves, distance between, how many more, how many fewer/less than, most, least, count back, how many left, how much less is, difference, count on, strategy, partition, tens, units, exchange, decrease, hundreds, value, digit.*

Year 4
Pupils should be taught to:

- Find 1000 less than a given number.
- Solve number and practical problems that involve subtraction with increasingly large positive numbers.
- Subtract numbers with up to 4 digits using the formal written methods of column subtraction where appropriate.
- Estimate and use inverse operations to check answers to a calculation.
- Solve two-step subtraction problems in contexts, deciding which operation to use and why.
- Subtract fractions with the same denominator.
- Count backwards in hundredths.
- Solve simple measure and money problems involving fractions and decimals to two decimal places.

Year 4 *Subtract with up to 4-digit numbers.*

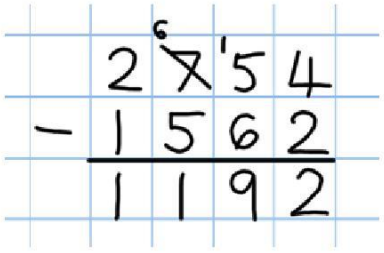
Partitioned column subtraction with 'exchanging' (decomposition):



As introduced in Y3, but moving towards more complex numbers and values. Use **place value counters** to reinforce 'exchanging'.

Subtracting money: partition into £1 + 30 + 5 for example.

Compact Column Subtraction:



To introduce the compact method, ask children to perform a subtraction calculation with the familiar partitioned column subtraction then display the compact version for the calculation they have done. Ask pupils to consider how it relates to the method they know, what is similar and what is different, to develop an understanding of it (shown on video).

Give plenty of opportunities to apply this to money and measures.

Always encourage children to consider the best method for the numbers involved— mental, counting on, counting back or written method (see video).

Mental Strategies:

A variety of mental strategies must be taught and practised, including counting on to find the difference, where numbers are closer together or where it is easier to count on.

Key Vocabulary: *equal to, take, less, minus, subtract, leaves, distance between, how many more, how many fewer/less than, most, least, count back, how many left, how much less is, difference, count on, strategy, partition, exchange, decrease, hundreds, value, digit, inverse.*

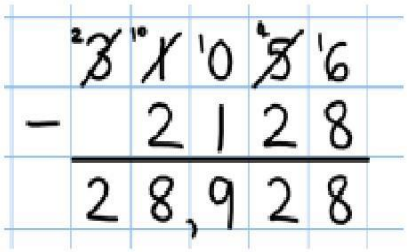
Year 5

Pupils should be taught to:

- Count backwards in steps of powers of 10 for any given number up to 1 000 000.
- Interpret negative numbers in context, count backwards with positive and negative whole numbers, including through zero.
- Subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction).
- Subtract numbers mentally with increasingly large numbers.
- Use rounding to check answers to calculations.
- Solve subtraction multi-step problems in contexts, deciding which operations and methods to use and why.
- Solve problems involving addition, subtraction, multiplication and division and a combination of these including understanding the meaning of the equals sign.
- Add fractions with the same denominator and denominators that are multiples of the same number.
- Solve problems involving measure, e.g. length, mass, volume, money using decimal notation, including scaling.
- Subtract fractions with the same denominator and denominators that are multiples of the same number.

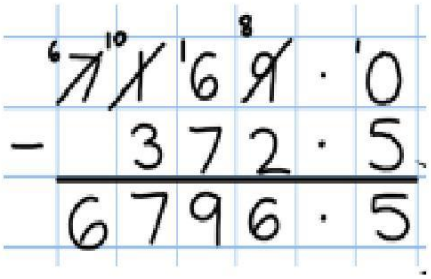
Year 5 *Subtract with at least 4-digit numbers including money, measures and decimals.*

Compact Column subtraction (with 'exchanging').



Children who are still not secure with number facts and place value will need to remain on the partitioned column method until ready for the compact method.

Subtract with decimal values, including mixtures of integers and decimals, aligning the decimal point.



Place a 'zero' in any empty decimal places to aid understanding of what to subtract in that column.

Create lots of opportunities for subtracting and finding differences with money and measures.

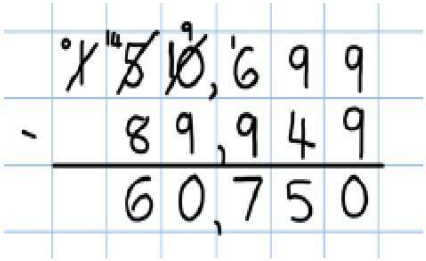
Key Vocabulary: *As for previous years + tenths, hundredths, decimal point, decimal.*

Year 6

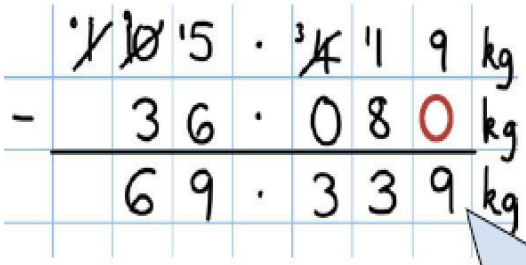
Pupils should be taught to:

- Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why.
- Solve problems involving addition, subtraction, multiplication and division.
- Practise addition, subtraction, multiplication and division for larger numbers, using the formal written methods of column addition and subtraction, short and long multiplication/division.
- Perform mental calculations including mixed operations and increasingly large numbers and more complex calculations, using a range of mental strategies.
- Subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions.
- Use simple algebraic formulae.
- Generate and describe linear number sequences.

Year 6 *Subtracting with increasingly large and more complex numbers and decimal values.*



Using the compact column method to subtract more complex integers



Using the compact column method to subtract money and measures, including decimals with different numbers of decimal places.

Empty decimal places can be filled with **zero** to show the place value in each column.

Key Vocabulary: *As for previous years.*

WRITTEN METHODS FOR MULTIPLICATION

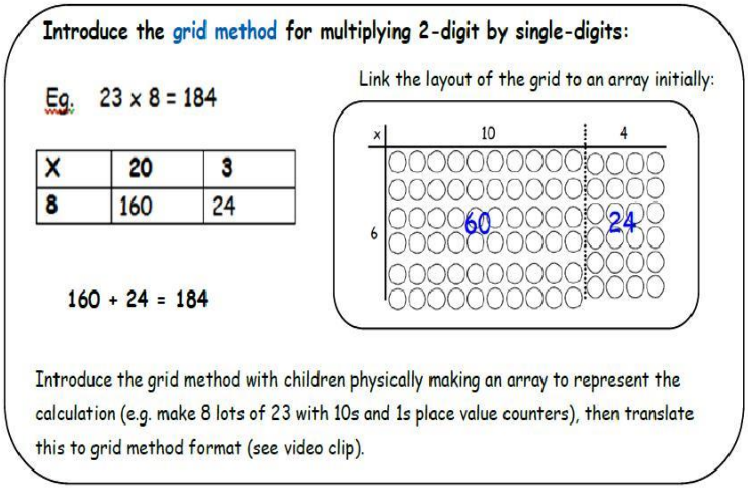
Year 3
Pupils should be taught to:

- Count from 0 in multiples of 4, 8, 50 and 100 and solve problems involving these ideas.
- Estimate the answer to a calculation using the inverse operations to check answers.
- Recall and use multiplication and division facts for the 3, 4 and 8 times tables.
- Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for 2-digit numbers x 1-digit numbers, using mental and progressing to formal written methods.
- Solve problems including missing number problems using multiplication and division.

Year 3 *Multiply 2-digits by a single digit number.*

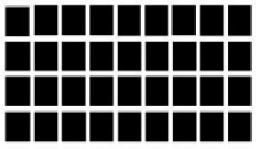
To multiply 2 digit number by a 1 digit number using the grid method.
38 x 7 = (30 x 7) + (8 x 7) = 210 + 56 = 266

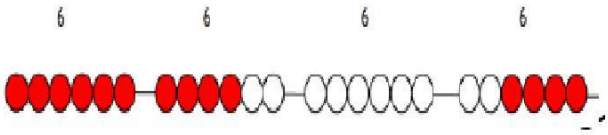
×	7
30	210
8	56
	266



To do this, children must be able to:

- Partition numbers into tens and units.
- Multiply multiples of ten by a single digit (e.g. 20 x 4) using their knowledge of multiplication facts and place value.
- Recall and work out multiplication facts in the 2, 3, 4, 5, 8 and 10 times tables.
- Work out multiplication facts not known by repeated addition or other taught mental strategies (e.g. by commutative law, working out near multiples and djusting, using doubling, etc.). Strategies to support this are repeated addition.
- Using a number line, bead bars and arrays:





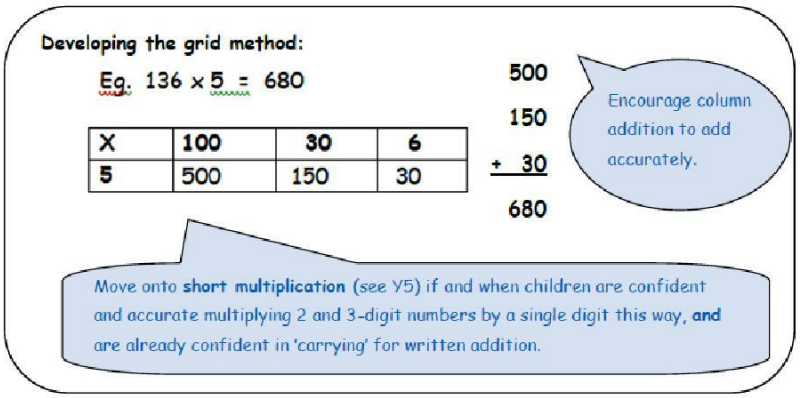
Key Vocabulary: *groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, commutative, sets of, equal groups, times, _ times as big as, once, twice etc., partition, grid method, multiple, product, tens, units, value.*

Year 4

Pupils should be taught to:

- Count in multiples of 6, 7, 9, 25 and 1000.
- Recall multiplication and division facts up to 12 x 12.
- Use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together 3 numbers.
- Recognise and use factor pairs and commutativity in mental calculations.
- Multiply 2-digit and 3-digit numbers by a one-digit number using formal written layout.
- Recognise that hundredths arise when dividing an object by 100 and dividing tenths by 10.
- Find the effect of dividing a 1 or 2-digit number by 10 and 100, identifying the value of the digits in the answer as ones, tenths or hundredths.
- Solve problems involving multiplying and adding, including using the distributive law to multiply 2-digit numbers by one digit, integer scaling problems and harder correspondence problems.

Year 4 *Multiply 2 and 3 digits by a single digit number, using all multiplication tables up to 12 x 12.*



Children should be able to:

- Approximate before they calculate, and make this a regular part of their calculating, going back to the approximation to check the reasonableness of their answer, e.g.

  346 x 9 is approximately 350 x 10 = 3 500

  Record an approximation to check the final answer against.

- Multiply multiples of ten and one hundred by a single-digit, using their multiplication table knowledge.
- Recall all times tables up to 12 x 12.

Key Vocabulary: *as for previous years + inverse.*

Year 5

Pupils should be taught to:

- Identify multiples and factors, including finding all factor pairs of a number and common factors of two numbers.
- Know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers.
- Establish whether a number up to 100 is prime and recall prime numbers up to 19.
- Multiply numbers up to 4 digits by a one or two-digit number using a formal written method, including long multiplication for two-digit numbers.
- Multiply numbers mentally drawing upon known facts.
- Multiply whole numbers and those involving decimals by 10, 100 and 1000.
- Recognise and use square numbers and cube numbers, and the²) notation for squared (and³) cubed.
- Solve problems involving multiplication including using their knowledge of factors and multiples, squares and cubes.
- Solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign.
- Solve problems involving multiplication, including scaling by simple fractions and problems.
- Multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams.

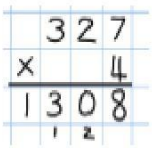
Year 5 *Multiply up to 4-digits by 1 or 2 digits.*

Introducing Column Multiplication:

- Introduce by comparing a grid method calculation to a short multiplication method, to see how the steps are related but emphasise how there are less steps involved in the column method.
- Children need to be taught to approximate first, e.g. for 72 x 38 they would use rounding (70 x 40 = 2 800) and use the approximation to check the reasonableness of their answer.
- Calculators may be used to check an answer after using approximation.

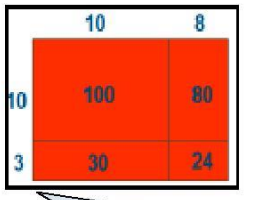
Short multiplication for multiplying by a single digit

x	300	20	7
4	1200	80	28



Pupils could be asked to work out a given calculation using the grid, and then compare it to 'your' column method. What are the similarities and differences? Unpick the steps and show how it reduces the steps.

Introduce long multiplication for multiplying by 2 digits

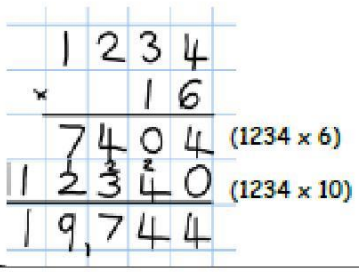


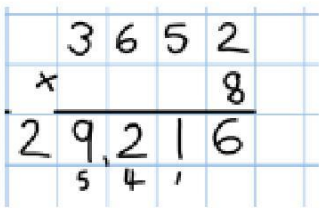
This grid could be used to introduce long multiplication, as the relationship can be seen in the answers in each row.

18 x 3 on the first row. (8 x 3 = 24, carrying the 2 for twenty, then '1' x 3)

18 x 10 on the 2nd row. Put a zero in the units first, then say 8 x 1 and 1 x 1.

Moving towards more complex numbers:





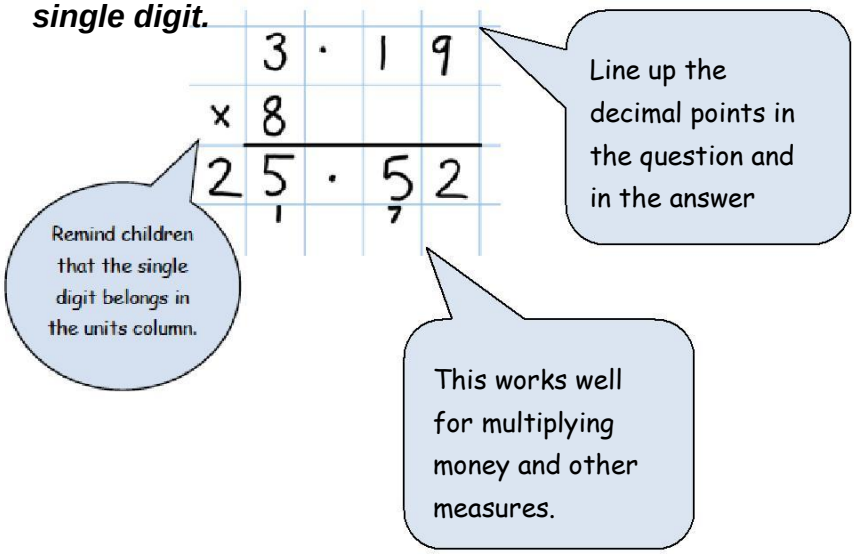
Key Vocabulary: As for previous years + factor, multiple, prime, square, cube.

Year 6

Pupils should be taught to:

- Multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication.
- Identify common factors, common multiples and prime numbers.
- Use their knowledge of the order or operations to carry out calculations involving the four operations.
- Solve problems involving the four operations.
- Use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy.
- Use common factors to simplify fractions and common multiples to express fractions in the same denomination.
- Multiply simple pairs of proper fractions, writing the answer in its simplest form.
- Multiply one-digit numbers with up to two decimal places by whole numbers.
- Solve problems involving the relative sizes of two quantities where missing values can be found by using integer multiplication and division facts.
- Solve problems involving the calculation and conversion of units of measure, using decimal notation up to 3 decimal places where appropriate.
- Use formulae for area and volume of shapes.

Year 6 ***Short and long multiplication as in Year 5, and multiply decimals with up to 2 digit place by a single digit.***



Key Vocabulary: As for previous years.

WRITTEN METHODS FOR DIVISION

Year 3

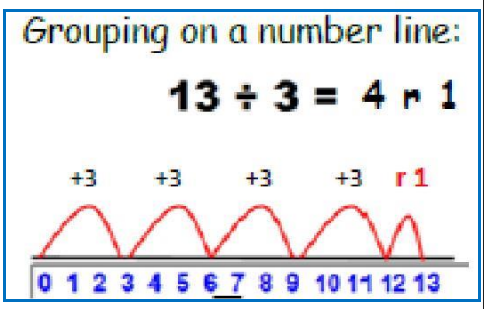
Pupils should be taught to:

- Recall and use multiplication and division facts for the **2, 3, 4, 5, 8 and 10** multiplication tables (through doubling, connect the 2, 4 and 8 tables).
- Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods.
- Solve problems, in contexts, and including missing number problems, involving multiplication and division.
- Develop efficient mental methods, for example, using multiplication and division facts (e.g. using 3 x 2 = 6, 6 ÷ 3 = 2, 2 = 6 ÷ 3) to derive related facts (30 x 2 = 60 so 60 ÷ 2 = 30 and 30 = 60 ÷ 2).
- Develop reliable written methods for division, starting with calculations of 2-digit numbers by 1-digit numbers and progressing to the formal written method of short division.
- Introduce the concept of remainders.

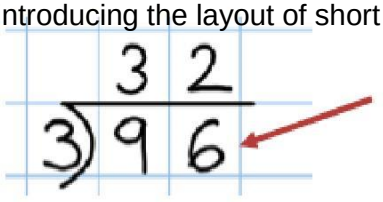
NB Real life contexts need to be used routinely to help pupils gain a full understanding, and the ability to recognise the place of division and how to apply it to problems.

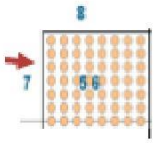
Year 3 *Divide 2-digit numbers by a single digit.*

Step 1: Children continue to work out unknown division facts by grouping on a number line from zero. They are also now taught the concept of remainders, as in the example. This should be introduced practically and with arrays, as well as being translated to a number line. Children should work towards calculating some basic division facts with remainders mentally for the **2s, 3s, 4s, 5s, 8s and 10s**, ready for 'carrying' remainders across within the short division method.



Step 2: Short division (limiting numbers to NO remainders in the answer or carried). Once children are secure with division as grouping and demonstrate this using number lines, arrays etc. short division for larger 2-digit numbers should be introduced, initially with carefully selected examples requiring no remainders at all. Start by introducing the layout of short division and comparing it to an array.

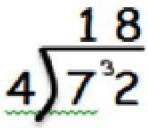




Remind children of correct place value, that 96 is equal to 90 and 6, but in short division, pose:

- How many 3's in 9? And record it above the 9 tens.
- How many 3's in 6? And record it above the 6 units.

Step 3:



Once children demonstrate a full understanding of remainders, and also the short division method, they can be taught how to use the method when remainders occur within the calculation (e.g. 96 ÷ 4), and be taught to 'carry' the remainder onto the next digit. If needed, children should use the number line to work out individual division facts that occur which they are not yet able to recall mentally.

Key Vocabulary: *share, share equally, one each, two each, group, equal of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, carry, remainder, multiple.*

Year 4

Pupils should be taught to:

- Recall multiplication and division facts for all numbers up to 12 x 12.
- Use place value, known and derived facts to multiply and divide mentally, including multiplying and dividing by 10 and 100 and 1.
- Practise to become fluent in the formal written method of short division with exact answers when dividing by a one-digit number.
- Pupils practise mental methods and extend this to three-digit numbers to derive facts, for example 200 x 3 = 600 so 600 ÷ 3 = 200.
- Pupils solve two-step problems in contexts, choosing the appropriate operation, working with increasingly harder numbers. This should include correspondence questions such as three cakes shared equally between 10 children.

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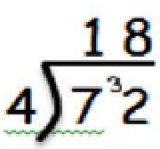
Year 4 *Divide up to 3-digit numbers by a single digit.*

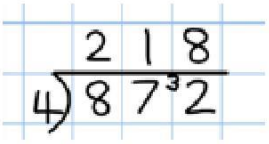
Short division should only be taught once pupils have secured the skill of calculating 'remainders'.

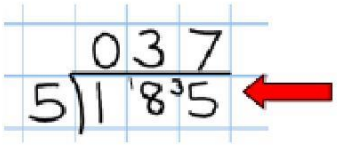


Step 1: Pupils must be secure with the process of short division for dividing 2 -digit numbers by a single digit (that do not result in a final remainder

– see Steps in Year 3), but must understand how to calculate remainders using this to 'carry' remainders within the calculation process.

Step 2: Pupils move onto dividing numbers with up to 3-digits by a single digit, however problems and calculations provided should not result in a final answer with a remainder at this stage. Children who exceed this expectation may progress to Yr 5 level.





Step 3: When the answer for the first column is zero (e.g. 1 ÷ 5) children could initially write a zero above to acknowledge its place, and must always 'carry' the number (1).

Key Vocabulary: *share, share equally, one each, two each, group, equal groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, carry, remainder, multiple, divisible by, factor.*

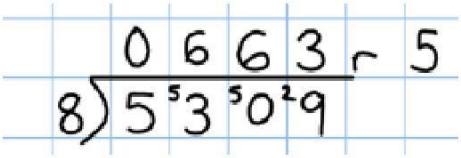
Year 5

Pupils should be taught to:

- Recall multiplication and division facts for all numbers up to 12 x 12 (as in Yr 4).
- Multiply and divide numbers mentally.
- Identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers.
- Solve problems involving multiplication and division where larger numbers are decomposed into their factors.
- Multiply and divide whole numbers and those involving decimals by 10, 100 and 1000.
- Use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers.
- Work out whether a number up to 100 is prime, and recall prime numbers to 19.
- Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context.
- Use multiplication and division as inverses.
- Interpret non-integer answers to division by expressing results in different ways according to the context, including with remainders, as fractions, as decimals or by rounding.
- Solve problems involving combinations of all four operations.

Year 5 ***Divide up to 4 digits by a single digit, including those with remainders.***

Short division with remainders: Now that pupils are introduced to examples that give rise to remainder answers, division needs to have a real life problem solving context, where pupils consider the meaning of the remainder and how to express it, e.g. as a fraction, decimal, or as a rounded number or value, depending upon the context of the problem.



- The answer to 5309 ÷ 8 could be expressed as 663 and five eighths, 663 r 5, as a decimal, or rounded as appropriate to the problem involved.
- See Yr 6 for how to continue the short division method to give a decimal answer for children who are confident.

If children are confident and accurate:

- Introduce long division for pupils who are ready to divide any number by a 2-digit number (e.g. 2678 ÷ 19). This is a Year 6 expectation.

Key Vocabulary: *As for previous years + quotient, prime number, prime factors, composite (non-prime) number.*

Year 6

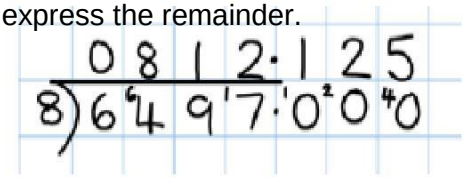
Pupils should be taught to:

- Recall and use multiplication and division facts for all numbers to 12 x 12 for more complex calculations.
- Divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions or by rounding, as appropriate for the context.
- Use short division.
- Perform mental calculations, including with mixed operations and large numbers.
- Identify common factors, common multiples and prime numbers.
- Solve problems involving all 4 operations.
- Use estimations to check answers to calculations and determine accuracy, in the context of a problem.
- Use written division methods in cases where the answer has up to two decimal places.
- Solve problems which require answers to be rounded to specified degrees of accuracy.

Year 6 ***Divide at least 4 digits by both single-digit and 2-digit numbers, including decimal numbers and quantities.***

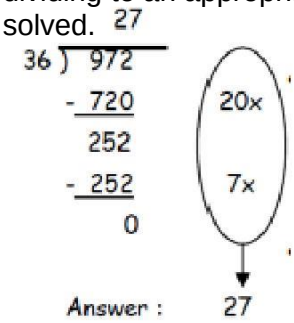
Short division, for dividing by a single digit, e.g. 6497 ÷ 8

Short division with remainders: Pupils should continue to use this method, but with numbers to at least 4 digits, and understand how to express remainders as fractions, decimals, whole number remainders, or rounded numbers. Real life problem-solving contexts need to be the starting point, where pupils have to consider the most appropriate way to express the remainder.



Calculating a decimal remainder:

In this example, rather than expressing the remainder as r1, a decimal point is added after the units because there is still a remainder, and the one remainder is carried onto zeroes after the decimal point (to show there was no decimal value in the original number). Keep dividing to an appropriate degree of accuracy for the problem being solved.



Introduce long division by chunking for dividing 2 digits:
Find out how many 36s there are in 972 by subtracting chunks of 36, until zero is reached (or until there is a remainder).
Teach pupils to write a 'useful list' first at the side that will help them decide what chunks to use, e.g. 1x = 36
10x = 360
100x = 3600

Key Vocabulary: *As for previous years.*

PROGRESSION IN MENTAL CALCULATION

Recall Children should be able to derive and recall:	**Mental Calculation Skills** Working mentally, with jottings to:	**Mental Methods or Strategies** Children should understand when to and be able to apply these strategies:
Year 3 • Use multiples of 2, 3, 4, 5, 8, 10, 50 and 100. • Multiplication and division facts for the 3, 4 and 8 multiplication tables. • Pairs of two-digit numbers with a total of 100, e.g. 32 + 68, or 32 + ? = 100. • Addition doubles of numbers 1 to 100, e.g. 38 + 38, and the corresponding halves. • Children should work towards calculating some basic division facts with remainders mentally for the **2s, 3s, 4s, 5s, 8s and 10s**.	• Add and subtract numbers mentally, including: ○ A three-digit number and units. ○ A three-digit number and tens. ○ A three-digit number and hundreds. • Solve problems in contexts, deciding which of the four operations to use and why, including measuring and scaling contexts, e.g. four times as high. • Estimate and read time with increasing accuracy to the nearest minute, including analogue clocks with roman numerals, 12 hour and 24 hour clocks. • Add and subtract amounts of money to give change.	• Apply partitioning related to place value using varied and increasingly complex problems e.g. 146 = 100 + 40 + 6 or 146 = 130 + 16. • Estimate the answer to a calculation and use inverse operations to check answers. • Solve problems, including missing number problems, using number facts, place value and more complex addition and subtraction. • Through doubling connect the 2, 4 and 8 multiplication tables e.g. 8 x 2 = double 4 x 2. • Develop efficient mental methods, e.g. using commutativity and associativity e.g. 4 x 12 x 5 = 4 x 5 x 12 = 20 x 12 = 240). • Subtract by counting up from the smaller to the larger number.
Year 4 • Count in multiples of 6, 7, 9, 25 and 1 000. • Recall multiplication and division facts for the multiplication tables up to 12 x 12. • Round any number to the nearest 10, 100 or 1000. • Round decimals with one decimal place to the nearest whole number. • Sums and differences of pairs of multiples of 10, 100 or 1000. • What must be added to any three-digit number to make the next multiple of 100, e.g. 521 + ? = 600. • Recognise families of equivalent fractions. • Pairs of fractions that total 1. • Percentage equivalents of one-half, one-quarter, three-quarters, tenths and hundredths.	• Solve number and practical problems with increasingly large positive numbers. • Solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why. • Add or subtract any pair of two-digit numbers, including crossing the tens and 100 boundary, e.g. 47 + 58, 91 – 35. • Add and subtract fractions with the same denominator. • Convert between different units of measure. • What must be added to any four-digit number to make the next multiple of 1000, e.g. 4087 + ? = 5000. • Solve problems involving converting from hours to minutes; minutes to seconds; years to months; weeks to days. • Find the difference between near multiples of 100, e.g. 607 – 588, or of 1000, e.g. 6070 –	• Estimate and use inverse operations to check answers to a calculation. • Use place value, known and derived facts to multiply and divide mentally, including multiplying by 0 and 1; dividing by 1, multiplying together three numbers. • Practise mental methods and extend to three-digit numbers to derive facts, e.g. 600 ÷ 3 = 200 can be derived from 2 x 3 = 6. • Combine knowledge of number facts and rules of arithmetic to solve mental and written calculations, e.g. 2 x 6 x 5 = 10 x 6 = 60. • Partition: count on or back in minutes and hours, bridging through 60 (analogue and digital times). • Relate area to arrays and multiplication.

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Recall Children should be able to derive	**Mental Calculation Skills** Working mentally, with jottings to:	**Mental Methods or Strategies** Children should understand when strategies:
Year 5 • Round any number up to 1 000 000 to the nearest 10, 100, 1 000, 10 000 and 100 000. • Round decimals with two decimal places to the nearest whole number and to one decimal place. • Sums and differences of decimals, e.g. 6.5 + 2.7, 7.8 – 1.3 • Doubles and halves of decimals, e.g. half of 5.6, double 3.4 • What must be added to a decimal with units and tenths to make the next whole number, e.g. 7.2 + ? = 8. • Squares×10 to 10. • Multiples and factor pairs of a number, and common factors of two numbers. • Multiply and divide mentally drawing upon known facts.	• Solve number and practical problems involving all key concepts. • Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why. • Practise mental calculations with increasingly large number to aid fluency. • Solve problems involving addition, subtraction, multiplication and division and a combination of these. • Solve problems involving number up to three decimal places. • Add or subtract a pair of two-digit numbers or three-digit multiples of 10, e .g. 38 + 86, 620 – 380, 350 + 360. • Add or subtract any pairs of decimal fractions each with units and tenths, e.g. 5.7 + 2.5, 6.3 – 4.8. • Add and subtract fractions with the same denominator and denominators that are multiples of the same number.	• Use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy. • Multiply and divide numbers mentally drawing upon known facts: apply all multiplication tables and division facts frequently, commit them to memory and use them confidently to make larger calculations. • Use multiplication and division as inverses to support the introduction of ratio in Year 6. • Make connections between percentages, fractions and decimals, e.g. 100% represents 1 whole and 1% is 1/100. • Count on or back in hundreds, tens, units and tenths • Subtract by counting up from the smaller to the larger number with larger numbers. • Use knowledge of place value and related calculations, e.g. 6.3 – 4.8 using 63 – 48. • Estimating volume and capacity.

Recall Children should be able to derive and recall:	**Mental Calculation Skills** Working mentally, with jottings to:	**Mental Methods or Strategies** Children should understand when to and be able to apply these strategies:
Year 6 • Round any whole number to a required degree of accuracy. • Addition and subtraction facts for multiples of 10 to 1000 and decimal numbers with one decimal place, e.g. 650 + ? = 930, ? – 1.4 = 2.5. • What must be added to a decimal with units, tenths and hundredths to make the next whole number, e.g. 7.26 + ? = 8. • Compare and order fractions. • Associate fractions with division and calculate decimal fraction equivalents, e.g. 0.375 for a simple fraction 3/8, 3 ÷ 8 = 0.375. • Simple formulae. • Squares to 12 x 12. • Prime numbers less than 100. • Equivalent fractions, decimals and percentages for hundredths, e.g. 35% is equivalent to 0.35 or 35100.	• Solve number and practical problems for all key concepts. • Perform mental calculations, including with mixed operations and large numbers. • Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why. • Undertake mental calculations with increasingly large numbers and more complex calculations. • Add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions. • Multiply simple pairs of proper fractions, writing the answer in its simplest form. • Solve problems involving the relative sizes of two quantities. • Multiply and divide two-digit decimals by whole numbers. • Find 10% or multiples of 10%, of whole numbers and quantities, e.g. 30% of 50 ml, 40% of £30, 70% of 200g.	• Use their knowledge of the order of operations to carry out calculations involving the four operations. • Use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy. • Relate common factors to finding equivalent fractions. • Use knowledge of place value and related calculations, e.g. 680 + 430, 6.8 + 4.3, 0.68 + 0.43 can all be worked out using the related calculation 68 + 43. • Partition: use partitioning and the distributive law to divide tens and ones separately, e.g. 92 ÷ 4 = (80 + 12) ÷ 4 = 20 + 3 = 23. • Form an equivalent calculation, e.g. to divide by 25, divide by 100, then multiply by 4; to divide by 50, divide by 100, then double. • Use knowledge of the equivalence between fractions and percentages and the relationship between fractions and division.